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He proceeds with the demonstrations, occupying two and one half pages of the *Quarterly*, avoiding the methods he criticizes and that by differentiation referred to in Mr. Finkel's problem, closing with the words "the results here obtained coincide, it may be remarked, with those given in Todhunter's *Int. Cal.*"

To solve Professor Finkel's problem we need only put $\alpha = 0$ in the first of the integrals mentioned by Walton.

437 (Calculus). Proposed by LEIGH PAGE, Yale University.

Integrate

$$\int_{-\infty}^{+\infty} \frac{\sin^2 x}{x^2} dx$$

without the use of the gamma functions.

I. SOLUTION BY OSCAR S. ADAMS, Coast and Geodetic Survey, Washington, D. C.

By direct integration, we have

$$\int_0^\infty e^{-ax} dx = \frac{1}{a}.$$

Let $a = \alpha - i\beta$. Then

$$\int_0^\infty e^{-ax+i\beta x} dx = \frac{1}{\alpha - i\beta} = \frac{\alpha + i\beta}{\alpha^2 + \beta^2},$$

or

$$\int_0^\infty e^{-ax} (\cos \beta x + i \sin \beta x) dx = \frac{\alpha + i\beta}{\alpha^2 + \beta^2}.$$

By equating real and imaginary parts we obtain the two definite integrals

$$\int_0^\infty e^{-ax} \cos \beta x dx = \frac{\alpha}{\alpha^2 + \beta^2}, \quad \text{and} \quad \int_0^\infty e^{-ax} \sin \beta x dx = \frac{\beta}{\alpha^2 + \beta^2}.$$

Since the first of these integrals is a uniform function of β , we have the relation

$$\int_0^\beta d\beta \int_0^\infty e^{-ax} \cos \beta x dx = \int_0^\infty dx \int_0^\beta e^{-ax} \cos \beta x d\beta = \int_0^\beta \frac{\alpha d\beta}{\alpha^2 + \beta^2};$$

or

$$\int_0^\infty e^{-ax} \frac{\sin \beta x}{x} dx = \tan^{-1} \frac{\beta}{\alpha}.$$

This integral is also a uniform function of β in the region $+\frac{\pi}{2} \geq \tan^{-1} \frac{\beta}{\alpha} \geq -\frac{\pi}{2}$.

Hence, in this domain, we have

$$\int_0^\infty dx \int_0^\beta e^{-ax} \frac{\sin \beta x}{x} d\beta = \int_0^\beta \tan^{-1} \frac{\beta}{\alpha} d\beta$$

or

$$\int_0^\infty \frac{e^{-ax}(1 - \cos 2x)}{x^2} dx = 2 \tan^{-1} \frac{2}{\alpha} - \frac{\alpha}{2} \log (\alpha^2 + 4).$$

This is a uniform function of α . Hence, α can converge to zero. This gives

$$\int_0^\infty \frac{2 \sin^2 x}{x^2} dx = 2 \cdot \frac{\pi}{2} = \pi, \quad \text{or} \quad \int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{1}{2}\pi.$$

Since $\sin^2 x/x^2$ is an even function of x , we have

$$\int_{-\infty}^\infty \frac{\sin^2 x}{x^2} dx = 2 \int_0^\infty \frac{\sin^2 x}{x^2} dx = \pi.$$

II. SOLUTION BY G. PAASWELL, N. Y. City.

Since $\sin^2 x/x^2$ is an even function of x we have